Chapter 1

The Math Behind Sherlock

1.1 Introduction

The mathematics behind determining the Point-of-Origin (PO) are based mainly in trigonometry and linear algebra. The following sections will provide the steps used by the BSA program on Sherlock to determine the PO from the provided blood stain data.

1.2 The Stains

All of the calculations used in BSA are based on the raw data collected from the stains themselves. The procedure used in the analysis follows directly from that described in Robert C. Shaler's book "Crime Scene Forensics" pages 388-390. The analyst must be diligent to minimize all sources of errors.

There are five important pieces of information which must be extracted from each and every stain thought to be parts of a unique pattern. All of these datum are subject to the interpretation of the shape of the blood stain.

1.2.1 The coordinates of the blood stain

It is important to record the relative position of all of the blood stains. In the current implementation of BSA, all of the blood stains are assumed to be from a single sample area which is flat and defines a single plane in three dimensions. BSA also assumes that the plane is perfectly vertical. A rectangular grid is defined on the surface which contains the stain. Each blood stain is examined to find a point within the grid where the first point of contact was made between each stain and the Y-Z surface. The Y measurement is taken from a reference plumb line, which defines the leftmost boundary of the target area, to the point of impact. The Z measurement is taken as the distance from the bottom of the target area to the stain. These two values are recorded as accurately as possible.

1.2.2 Width and Length

The next two parameters to measure are the length and width of the droplet itself. Once an approximate outline of the stain is obtained, the analyst will measure the lengths of the major and minor axes of the stain. A representation of this can be seen in Figure 14.11 in Shaler's book (Page 388).

The ratio of these two measurements will be used to determine the angle of impact of the blood droplet as it hit the surface. Equation 1.1 is used to convert these to measurements into the impact angle

$$\alpha = \arcsin\left(\frac{\text{Width of stain}}{\text{Length of stain}}\right) \tag{1.1}$$

This value for α will be used later to determine the distance, X, "above" the surface containing the blood stains, of the point of origin.

1.2.3 The direction angle: γ

The last measurement required from the analyst is that of the direction of travel of the individual blood spatter droplets. Each stain on the surface will come from individual droplets travelling in unique directions from the source. By measuring the directionality of the stains, a common point of convergence can be determined. When this exercise is conducted manually, technicians can use physical "strings" to represent directionality.

The Sherlock BSA program requires a measurement of the angle made between the long axis of the blood stain and a standard reference direction. We will call this angle γ. This standard reference direction needs to be consistent from stain to stain. This allows for the evaluation of the relative directions between droplets. This leads to the ability to determine a common point of convergence. (See Figure 14.12 in Shaler's book. Page 389).

In the datasets presented to Sherlock's BSA application, the standard reference direction is a vertical line. The vertical line passes through the tip of the blood stain closest to the point of impact, where the droplet first made contact with the surface. The coordinate of this point were recorded as the first two datum above. The current convention is that angles are measured in a clockwise direction between the vertical reference line and the long axis of the stain. This is contrary to how angles are taught to be measured in most geometry classes. Typically angles are measured relative to a horizontal line in a counter-clockwise fashion. This method of measuring the angle is consistent with the conventions expected in spherical polar coordinate systems (See: http://mathworld.wolfram.com/SphericalCoordinates.html)

1.3 The Area-of-Origin: AO

The point of origin, or the source of the blood spatter, can be determined using the data described in the previous section. Combining the information from multiple blood spatter stains allows for multiple evaluations of the AO. Combining these results permits the calculation of an average point in 3D space where the impact occurred.

By taking the direction of travel of two blood spatter stains, we can back track the trajectories and find where the two lines intersect. We can do this for each pair of blood stain trajectories. Following the work of Carter (2001), intersections between trajectories were constrained to pairs of stains selected from opposite sides of the pattern. By this we mean that each stain on the left hand side of the pattern is only matched with stain trajectories from the right hand side. This alleviates the possibility that two stains on the same side of the pattern are actually related in some way. The collection of trajectory intersections gives us a cloud of points which represent an approximation of a common point of convergence. The more lines there are, the better the average coordinates of the point of intersection will approximate the actual value.

When ejected from the area of origin, blood droplets follow a ballistic trajectory until they impact on the surface we are analysing. The forces acting on the droplets while they are in flight are due to air resistance and gravity. The Y and Z coordinates that we measure therefore are affected by the curvature of the droplet's path. This in turn can affect the value of the impact angle α. This will have the effect of giving us a slightly elevated value for α which can lead to an over estimation of the height of the area of origin. To minimize this impact on the evaluation of the area of origin, we will first evaluate the height of the impact: X.

In flight, droplets are not subjected to any lateral forces which might see them veer to the right or to the left. If we were to observe the droplets in flight from a vantage point well above the scene, they would trace a straight line between the area of origin and the target area. We can use this straight line trajectory and the angle α to determine an accurate value for the X coordinate of the area of origin.

For each individual line, we know that the point of impact of the stain lies in the Y-Z plane. We also know the impact angle α. We can use these values to project the flight path of the blood droplet onto the X-Y plane. Using the base Y coordinate given in the data, we can develop a parametric equation for each trajectory. A parametric equation is just a description of the direction and rate of change of the X and Y values for each individual line.

1.4 Calculating the point of intersection

To find the point of intersection of two lines, we must first construct the equation for a line on a plane. As we can see from figure 1.1, we can calculate the coordinates of the point Q if we know the coordinates of the point P as well as the characteristic values for ΔX and ΔY for this specific line. The ratio of ΔY over ΔX is commonly know as the slope of the line. We can determine these values based on the value of α determined by the technician. From trigonometry we typically express ΔX as relating to the cosine of the angle α and ΔY, the sin.

For many purposes, it is sufficient to express the equation of a line as:

$$y = \frac{\Delta y}{\Delta x} x + b \tag{1.2}$$

Where b is the Y axis intercept when x=0.

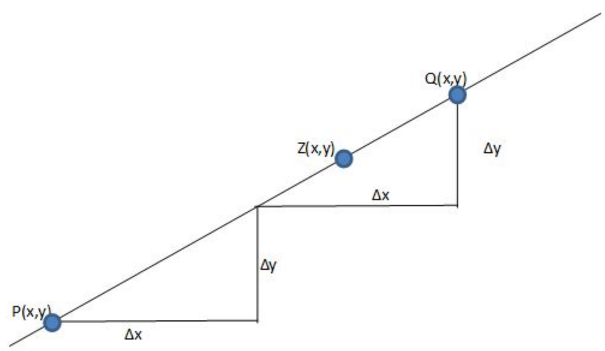

Figure 1.1: A line on a plane.

A more natural way of representing the equation of the line is to recognize that: knowing the coordinates of one point on the line, we can add an identical number of ΔX to the X coordinates and ΔY to the Y coordinate to find another point on the line. In the example shown in figure 1.1, we can write the coordinates for the point Q as:

$$\begin{aligned} x_Q &= x_P + 2\Delta X \\ y_Q &= y_P + 2\Delta Y \end{aligned} \tag{1.3}$$

This bring us to a more generic method for representing a line know as a parametric equation. The two representations are equivalent but in the parametric form, the different coordinates are represented using separate equations. For a line in a 2D plane, the parametric equation looks like:

$$\begin{aligned} x &= x_0 + ta \\ y &= y_0 + tb \end{aligned} \tag{1.4}$$

Where, in this representation a and b are calculated as the differences in X or Y coordinates of any two points on the line:

$$\begin{aligned} a &= \Delta x = (x_1 - x_0) \\ b &= \Delta y = (y_1 - y_0) \end{aligned} \tag{1.5}$$

In the Sherlock implementation of BSA, the technician supplies the code with a single coordinate for each blood stain. The software then uses that value as well as the angle α to generate a second point which lies on the same line. We can then create parametric equations for the trajectory of each droplet generating each blood stain. Our next task is to find the point of intersection for each pair of lines.

For this we will need the parametric equations for two lines. This will require 2 points defined per line. A representation of this is given in Figure 1.2.

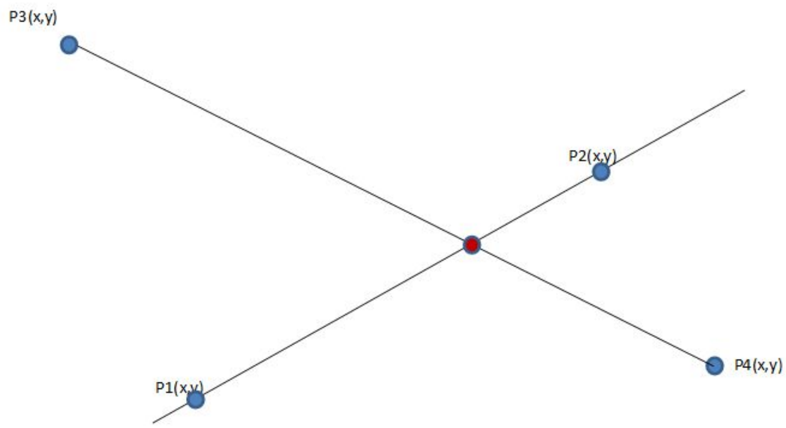

Figure 1.2: Two lines and their point of intersection (red).

Line 1 passes through the points P1 and P2. The coordinates for each will be represented as (x_1, y_1) and (x_2, y_2) respectively. Similarly points P3 and P4 are (x_3, y_3) and (x_4, y_4). Their parametric equations are then:

$$\begin{aligned} x &= x_1 + t_1(x_2 - x_1) \\ y &= y_1 + t_1(y_2 - y_1) \end{aligned} \tag{1.6}$$

Line 2:

$$\begin{aligned} x &= x_3 + t_2(x_4 - x_3) \\ y &= y_3 + t_2(y_4 - y_3) \end{aligned} \tag{1.7}$$

At the point of intersection, the x and y coordinates are equal for both equations. We can use this property to calculate the coordinate where the two lines meet. Combining the equations for the value of x from 1.6 and 1.7, yields:

$$\begin{aligned} x_1 + t_1(x_2 - x_1) &= x_3 + t_2(x_4 - x_3) \\ x_1 + t_1(x_2 - x_1) - x_3 &= t_2(x_4 - x_3) \\ t_2 &= \frac{(x_1 - x_3) + t_1(x_2 - x_1)}{(x_4 - x_3)} \end{aligned} \tag{1.8}$$

We now have an expression for t_2 in terms of a number of variables for which we know the value except for one: t_1. We can get a different expression for t_2 value by using the equations for the y coordinates at the point of intersection.

$$\begin{aligned} y_1 + t_1(y_2 - y_1) &= y_3 + t_2(y_4 - y_3) \\ y_1 + t_1(y_2 - y_1) - y_3 &= t_2(y_4 - y_3) \\ t_2 &= \frac{(y_1 - y_3) + t_1(y_2 - y_1)}{(y_4 - y_3)} \end{aligned} \tag{1.9}$$

Combining these based on the equivalence of t_2 gives us:

$$\frac{(x_1 - x_3) + t_1(x_2 - x_1)}{(x_4 - x_3)} = \frac{(y_1 - y_3) + t_1(y_2 - y_1)}{(y_4 - y_3)} \tag{1.10}$$

Manipulating these equations and solving for t_1 yields:

$$t_1 = \frac{\left[\frac{(x_1 - x_3)}{(x_4 - x_3)} - \frac{(y_1 - y_3)}{(y_4 - y_3)}\right]}{\left[\frac{(y_2 - y_1)}{(y_4 - y_3)} - \frac{(x_2 - x_1)}{(x_4 - x_3)}\right]} \tag{1.11}$$

We can now use this value for t_1 to back-substitute in the equation 1.6 and determine the x and y coordinates of the point of intersection.

Looking at equation 1.11, it is important to observe that there are a number of conditions which would make this equation impossible to resolve. Should the denominator in any of these relations equate to zero, we would not be able to solve for a valid value for t_1. Should any of the two lines be vertical in the plane, these equations would fail. In the software on Sherlock this special case is taken into account by realizing that the Y coordinate of the vertical trajectory remains unchanged. We can therefore easily solve the parametric equation for the second trajectory for this specific value of Y to yield an appropriate value for t_2.

It is important to remember that such a point of intersection exists for each pair of lines. Each pair of blood stains will generate their own unique point of intersection using this method. If no errors were present in the measurement data, then all of these points of intersection should be identical. Due to the nature of the scene and measurement tools, some errors will affect the results we obtain. Each intersection point however, is to the best of the technicians ability, as accurate as possible. By combining these values together, we obtain a group estimate for the actual point of intersection of all of these lines. By doing this, we introduce more uncertainty into our answer while at the same time, attempting to minimize the effects of outlier data points.

1.5 Height of the Point of Origin

Once a common point of intersection has been determined in the X-Y plane, we have determined the distance between the point of impact which generated the stains and the surface upon which we have made our measurements. We can turn our focus to the determination of the third coordinate for the point of origin of the blood stains, the Z coordinate or the height of the AO.

In the previous section we have determined both the X and Y coordinates of the AO which best represent the average values for all trajectories. We have developed parametric equations for each of the droplet trajectories. Determining the Z value for the AO consists of finding a t value for each line equation which generates the appropriate X,Y values. We then use this t value to generate a Z value belonging to each trajectory. As with the case of averaging out the X,Y coordinates, the collection of Z values is averaged to determine the Z coordinate of the AO.

At this point in the analysis, we have used the field data to generate all three coordinates for the AO.

1.6 conclusion

This document lists the number of steps required to translate field measurements of a collection of blood stains, to an approximation of the position of the Point of Origin. All of the steps required are based in geometry and some knowledge of trigonometric functions. Hopefully, this document has proven helpful.

by Jacques Béland 2018

TRENT UNIVERSITY

Peterborough, Ontario, Canada